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NUMERICAL METHODS FOR WATER-WAVE RADIATION PROBLEMS

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SUMMARY

A boundary integral equation method is used to compute the forces acting on bodies oscillating at or near the free surface of a fluid. This method relies on the use of a Green function representing the potential of a unit pulsating source beneath the free surface. A peculiarity of the boundary integral equation method in the presence of a free surface is that it breaks down at certain characteristic frequencies also known as 'irregular frequencies'. The objectives of the present study are to develop simple and efficient algorithms for the numerical evaluation of the Green function as well as a method for removing the irregular frequencies. In particular, systematic expansions for the Green function have been developed. These algorithms are very efficient and remarkably simple to implement numerically. In order to remove the irregular frequencies, the boundary integral equation is supplemented by the first N null-field equations. As a result the first N irregular frequencies are eliminated. This method introduces a very small additional computational cost and is general enough so that it can be applied to any free surface problem.

KEY WORDS Boundary integral equation method Green function Irregular frequencies

1. INTRODUCTION

During the last 20 years there has been growing interest in methods for calculating the wave loads on ships and offshore platforms and predicting their unsteady motions. The first analyses were limited to bodies of simple geometry such as submerged circular cylinders, spheres or ellipsoids. Another special case of body shape that has been studied extensively is that of a slender ship using methods from perturbation theory. Despite its utility, this analysis is limited; it produces poor results close to the bow and stern of a ship and cannot be used for typical offshore structures. The advent of large, powerful computers has led to the development of numerical methods that removed the geometrical restrictions of the earlier methods. In recent years a number of numerical methods for calculating wave diffraction and radiation involving three-dimensional bodies of arbitrary shape have been developed.

The most popular of these methods is the boundary integral method. This approach is based on Green's theorem with an appropriate Green function and has the major advantage of reducing the problem to solving an integral equation on the fluid boundaries and therefore the dimensions of the computational domain are reduced by one. The integral equation is solved numerically using a panel method. Hess and Smith[1] were the first to develop the method to a practical stage for aerodynamic applications for an infinite fluid. Following their analysis, the body surface is approximated by a number of plane quadrilateral elements of constant singularity strength and the integral equation is solved by collocation, resulting in a linear system of equations for the singularity strengths.

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The main numerical task in extending the method to free surface problems is to compute the Green function efficiently and accurately. The Green function in this case, which represents the potential of a unit pulsating source beneath the free surface, is a substantially more complicated function than that encountered in infinite fluid, elastodynamics or acoustics problems. For most practical cases the body surface must be approximated by at least 100 panels and the computations must be performed for at least 10 frequencies. Since the Green function and its derivatives must be evaluated for each combination of panels and at each frequency, under these assumptions at least 10^5 Green function evaluations are required. Therefore a peculiarity of this method compared to a more traditional finite element method is that the most important numerical task is in evaluating the coefficients of the resulting linear system rather than solving it.

The fundamental importance of the Green function for this problem has motivated several studies on its numerical calculation. A survey of different integral representations and some series expansions may be found in Reference 2. In Reference 3 numerical integration was used to obtain tables of the values of the Green function and its derivatives. These tables are accurate up to 15 decimal places and very useful as a benchmark for checking the accuracy of numerical algorithms. However, for most cases of practical interest, numerical integration cannot be used because of the large computational time involved. A more attractive alternative is to develop power series and asymptotic series expansions. Examples of such studies include Reference 4 as well as Reference 5. In Reference 6 the use of Chebychev expansions to evaluate the Green function is outlined. However, no specific details are given since these expansions are contained in the commercial code FINGREEN.

Another complication of the boundary integral method in the presence of a free surface is that it does not have a unique solution at certain characteristic frequencies also known as 'irregular frequencies'. These frequencies correspond to eigenvalues where the interior problem has a non-trivial solution. This phenomenon was first pointed out by John[7] and verified numerically by Frank[8] for oscillating cylinders in two dimensions. Large numerical errors occur in a frequency bandwidth close to each irregular frequency and this phenomenon has a severe effect especially in the high-frequency range where the irregular frequencies are closer to each other. The existence of the irregular frequencies represents the most serious drawback of the boundary integral equation method and several methods for removing them have been proposed.[9-14]

The objectives of the present study are to develop simple and efficient algorithms for the numerical evaluation of the Green function as well as a method for removing the irregular frequencies. In particular, systematic expansions for the Green function have been developed. Depending on the values of the arguments, four different expansions are used. One expansion consists of a Taylor series, another is a numerical form based on the Haskind integral representation, while the other two are series involving special functions. No special function evaluation is needed; in all cases the special function values are obtained by using recursion formulae. All these algorithms are very efficient and remarkably simple to implement numerically. In order to remove the irregular frequencies, the boundary integral equation is supplemented by the first N null-field equations. The resulting overdetermined system is then solved using a least squares procedure. As a result the first N irregular frequencies are eliminated. This method introduces a very small additional computational cost and is general enough so that it can be applied to any free surface problem.

A computer program has been developed. Its efficiency and accuracy are illustrated by computing the added mass and damping of a sphere and a right circular cylinder in heave and surge. In all cases considered the results agree well with other published results and are free from the irregular frequency problem.

2. MATHEMATICAL FORMULATION

2.1. *The boundary value problem*

Consider a rigid body floating on the free surface of deep water and performing forced harmonic oscillations of small amplitude at a radian frequency ω. An xyz-co-ordinate system is defined with z positive upwards and the xy-plane coincident with the calm free surface. Under the usual assumptions of an inviscid, irrational flow the problem may be described by a velocity potential $\Phi(x, y, z, t)$. It is convenient to write Φ as

$$\Phi(x, y, z, t)=\mathrm{Re}\left(\sum_{j=1}^{6} \phi_j(x, y, z)\xi_j e^{-i\omega t}\right), \tag{1}$$

with ξ_j the complex amplitude of the body motion in each of the six degrees of freedom of the body.

In the fluid domain each of the potentials must satisfy the Laplace equation

$$\nabla^2\phi_j=0. \tag{2}$$

The boundary condition on the free surface in its linear form is

$$-\nu\phi_j+(\phi_j)_z=0 \quad \text{on } z=0, \tag{3}$$

where $\nu=\omega^2/g$ is the wave number.

On the body surface S each of the radiation potentials must satisfy

$$\frac{\partial\phi_j}{\partial n}=-i\omega n_j=V_j(P), \quad j=1, 2, \ldots, 6, \tag{4}$$

where n_j are the components of the generalized normal directed out of the fluid domain:

$$(n_1, n_2, n_3)=\mathbf{n}, \qquad (n_4, n_5, n_6)=\mathbf{r}\times\mathbf{n}, \qquad \mathbf{r}=(x, y, z).$$

In addition to the above, appropriate radiation conditions at infinity are needed in order to make the solution unique.

2.2. *Derivation of the integral equation*

The Green function for this problem, which is the potential of a submerged, pulsating source, is given by Wehausen and Laitone (Reference 15, equation (13.17)) as $\mathrm{Re}[G(P, Q)e^{-i\omega t}]$ with

$$G(P, Q)=\frac{1}{r}+\frac{1}{r'}+2\int_0^\infty \frac{e^{k(z+\zeta)}}{k-\nu}J_0(kR)\,dk+2\pi\nu i\, e^{\nu(z+\zeta)}J_0(\nu R), \tag{5}$$

where $P=(x, y, z)$ is the field point, $Q=(\xi, \eta, \zeta)$ is the source point,

$$r^2=(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2,$$
$$r'^2=(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2,$$
$$R^2=(x-\xi)^2+(y-\eta)^2,$$

$\nu=\omega^2/g$ is the wave number, J_0 is the Bessel function of the first kind and $\int$ denotes the principal value.

Application of Green's theorem provides a Fredholm integral equation of the second kind for the values of the potential on the body surface S:

$$2\pi\phi_j(P)+\int_s\int \phi_j(Q)\frac{\partial G(P,Q)}{\partial n_Q}\,\mathrm{d}S_Q=\int_s\int V_j(Q)G(P,Q)\,\mathrm{d}S_Q. \tag{6}$$

From the values of the potential on the body surface the added mass and damping coefficients may be obtained. Following the standard definition given in Reference 16, the added mass and damping coefficients are

$$A_{ij}=\mathrm{Re}\left[\left(-\rho\int_s\int \phi_j(Q)n_i(Q)\,\mathrm{d}S_Q\right)\Big/\,\mathrm{i}\omega\right] \quad \text{and}$$

$$B_{ij}=\mathrm{Re}\left(\rho\int_s\int \phi_j(Q)n_i(Q)\,\mathrm{d}S_Q\right), \quad i,j=1,2,\ldots,6. \tag{7}$$

2.3. The discretized problem

The integral equation (6) is solved numerically using a panel method. The body surface is approximated by an ensemble of flat quadrilateral panels. The value of the potential is assumed constant on each panel, reducing the problem of finding a continuous potential distribution to determining a finite number of unknown potential strengths. These potential strengths are determined by collocation, where the integral equation (6) is satisfied at one point for each panel. Following this procedure, equation (6) may be written in the discrete form

$$\sum_{m=1}^{M} A_{nm}(\phi_j)_m=B_n, \quad n=1,2,\ldots,M, \tag{8}$$

where M is the number of quadrilateral elements, $(\phi_j)_m$ is the potential strength of the mth element,

$$A_{nm}=\begin{cases}\displaystyle\int_{s_m}\int\frac{\partial G(P_n,Q)}{\partial n_Q}\,\mathrm{d}S_Q, & n\neq m,\\ 2\pi, & n=m,\end{cases}$$

$$B_n=\sum_{m=1}^{M}(V_j)_m\int_{s_m}\int G(P_n,Q)\,\mathrm{d}S_Q.$$

A 2×2 product Gauss quadrature rule is used to evaluate the integrals of the Green function and its derivatives over each panel.

2.4. Null-field equations

A different approach to solve the boundary value problem for ϕ was proposed by Martin.[17,18] A point in the interior of the body is selected and designated as the origin. Then the Green function G may be expanded as

$$G(P,Q)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{\sigma=1}^{2}\alpha_{mn}^{\sigma}(P)\Phi_{mn}^{\sigma}(Q), \tag{9}$$

where the water wave multipoles $\Phi_{mn}^{\sigma}(Q)$ are a complete set of harmonic potentials satisfying the free surface condition (3) and $\alpha_{mn}^{\sigma}(P)$ are regular coefficients. Both $\alpha_{mn}^{\sigma}(P)$ and $\Phi_{mn}^{\sigma}(Q)$ are defined in Reference 17.

This approach leads to an infinite set of moment-like equations called the null-field equations for water waves:

$$\int_s \int \left(\phi_j(Q) \frac{\partial \Phi^{\sigma}_{mn}(Q)}{\partial n_Q} - V_j(Q) \Phi^{\sigma}_{mn}(Q) \right) \mathrm{d}S_Q = 0, \quad \sigma = 1, 2, \quad m, n = 0, 1, 2, \ldots . \tag{10}$$

It is possible to show that the null-field equations possess a unique solution at all frequencies.[17] Furthermore, the null-field equations have been used to solve radiation problems in two dimensions involving bodies of simple geometry such as heaving circular and elliptical cylinders. However, their use appears to be limited to bodies of simple geometrical shape.

3. EVALUATION OF THE GREEN FUNCTION

The task here is to evaluate the principal value integral term, which, following Newman,[6] may be written as a function of two non-dimensional, non-negative parameters $X = \nu[(x-\xi)^2 + (y-\eta)^2]^{1/2}$ and $Y = -\nu(z+\zeta)$.

We have

$$G(P, Q) = \frac{1}{r} + \frac{1}{r'} + \nu F(X, Y) + 2\pi \mathrm{i} \nu \mathrm{e}^{-Y} J_0(X), \tag{11}$$

with

$$F(X, Y) = 2 \int_0^{\infty} \frac{\mathrm{e}^{-kY}}{k-1} J_0(kX) \, \mathrm{d}k.$$

The integral $F(X, Y)$ can also be transformed into a finite non-singular integral,[19]

$$F(X, Y) = -\pi \mathrm{e}^{-Y} (Y_0(X) + H_0(X)) - 2g(X, Y), \tag{12}$$

where the finite integral term $g(X, Y)$ is given by

$$g(X, Y) = \int_0^Y \frac{\mathrm{e}^{t-Y}}{\sqrt{(X^2 + t^2)}} \mathrm{d}t. \tag{13}$$

Here Y_0 is the Bessel function of the second kind and H_0 is the Struve function.

The Bessel functions J_0 and Y_0 and the Struve function H_0 involved in equations (11) and (13) may be evaluated numerically using efficient approximations given in References 20 and 21.

Equation (13) provides a decomposition of the Green function into a far-field term which represents waves radiating to infinity and a non-oscillatory near-field term $g(X, Y)$. This near-field term $g(X, Y)$ represents a local disturbance decaying monotonically for increasing X. It is therefore advantageous to use the decomposition (12) for large values of the ratio X/Y. The form (11) is more useful for small X/Y.

3.1. Expansion for small X and Y

The near-field term $g(X, Y)$ may be expanded in a Neumann series involving products of Bessel functions with hyperbolic cosines which converges for small values of X and Y. If we change the variable of integration in (13) by setting $u = \sinh^{-1}(t/X)$, we obtain

$$g(X, Y) = \int_0^{\sinh^{-1}(Y/X)} \mathrm{e}^{X \sinh u - Y} \mathrm{d}u. \tag{14}$$

Making use of the fact that $e^{X(t-1/t)/2}$ is the generating function for the Bessel function, we may write

$$e^{X\sinh u}=J_0(X)+\sum_{n=1}^{\infty}[e^{nu}+(-1)^n e^{-nu}]J_n(X).$$

Then

$$g(X, Y)=\int_0^{\sinh^{-1}(Y/X)}\left(J_0(X)+2\sum_{n=1}^{\infty}\{\cosh(2nu)J_{2n}(X)+\sinh[(2n-1)u]J_{2n-1}(X)\}\right)du,$$

with the final result

$$g(X, Y)=e^{-Y}\left[vJ_0(X)+2\sum_{n=1}^{\infty}\left(\frac{\sinh(2nv)}{2n}J_{2n}(X)+\frac{\cosh(2n-1)v-1}{2n-1}J_{2n-1}(X)\right)\right], \quad (15)$$

where

$$v=\sinh^{-1}\left(\frac{Y}{X}\right)=\ln\left[\frac{Y}{X}+\left(1+\frac{Y^2}{X^2}\right)^{1/2}\right].$$

For large orders we have

$$J_n(X)\sim\frac{1}{\sqrt{(2\pi n)}}\left(\frac{eX}{2n}\right)^n;$$

therefore for sufficiently small values of X and Y the Neumann series (15) converges after only a few terms.

Algorithm (15) is remarkably simple to implement numerically. The Bessel functions $J_n(X)$ are evaluated using the algorithm described in Reference 20 (equation (9.12)). Briefly, this algorithm may be summarized as follows. Pick up arbitrary starting values for $J_{n+1}(X)=0$ and $J_n(X)=1$. Use the recurrence relation to evaluate $J_{n-1}(X), J_{n-2}(X), \ldots, J_0(X)$. Normalize the results by use of the equation

$$J_0(X)+2J_2(X)+2J_4(X)+\ldots=1.$$

This procedure has been implemented to evaluate the terms in the Neumann series (15). It has the attractive feature that no Bessel function evaluation is necessary.

Equation (15) may be used to evaluate the Green function with 6D accuracy in the domain defined by $0\leq Y\leq 2$, $0{\cdot}5Y\leq X\leq 5$. For values of $X\leq 0{\cdot}5$ the number of terms required is $4+11X$ with a maximum of nine terms. For values of $X\geq 0{\cdot}5$ the series may be truncated after 15 terms.

3.2. Expansion for large X

When the value of X is large, the near-field term $g(X, Y)$ may be expanded in a series involving products of the Legendre polynomials and the incomplete gamma function. We change the variable of integration in (13) by setting $\tau=(Y-t)/R_1$, where $R_1=\sqrt{(X^2+Y^2)}=vr'$, with the result

$$g(X, Y)=\int_0^{\cos\theta}e^{-\tau R_1}\left[\frac{X^2}{R_1^2}+\left(\frac{Y}{R_1}-\tau\right)^2\right]^{-1/2}d\tau$$

$$=\int_0^{\cos\theta}e^{-\tau R_1}(1-2\tau\cos\theta+\tau^2)^{-1/2}d\tau, \quad (16)$$

where the ratio

$$\frac{Y}{R_1}=-\frac{z+\zeta}{r'}=\cos\theta$$

in spherical co-ordinates. Since $(1-2\tau\cos\theta+\tau^2)^{-1/2}$ is the generating function for Legendre polynomials, we may write

$$(1-2\tau\cos\theta+\tau^2)^{-1/2}=\sum_{n=0}^{\infty} P_n(\cos\theta)\tau^n,$$

where P_n is the Legendre polynomial of order n. We thus obtain

$$g(X, Y)=\sum_{n=0}^{\infty} P_n(\cos\theta)\int_0^{\cos\theta} e^{-\tau R_1}\tau^n\,d\tau$$

$$=\sum_{n=0}^{\infty}\frac{P_n(\cos\theta)\gamma(n+1, Y)}{R_1^{n+1}}, \tag{17}$$

where $\gamma(n+1, Y)$ is the incomplete gamma function which is defined as (Reference 20, equation (6.5.2)) $\gamma(n+1, Y)=\int_0^Y du\, e^{-u}u^n$.

For large values of Y, $\gamma(n+1, Y)\to n!$ and formula (9) gives

$$g(X, Y)=\sum_{n=0}^{\infty}\frac{n!\,P_n(\cos\theta)}{R_1^{n+1}} \tag{18}$$

This limiting case of the general formula (18) was derived by Newman[6] by repeated integration by parts.

Algorithm (17) can be used for efficiently evaluating the function $g(X, Y)$. The Legendre polynomials and the incomplete gamma functions of any order n may be determined from the forward recursion relations (Reference 20, equations (8.5.3) and (6.5.22)), where the starting values $P_0(\cos\theta)=1$, $P_1(\cos\theta)=\cos\theta$ and $\gamma(1, Y)=1-e^{-Y}$ are used. The forward recursion relations are stable for the number of terms used. Again no special function evaluations are needed.

Equation (17) may be used in the domain defined as shown in Figure 1 by $X\geq 5$. The number of terms required for 6D accuracy varies from three to 13 and is given in Table I for different values of X and Y.

3.3. Expansion for small X/Y

For small values of the ratio X/Y the Bessel function in the integral representation (11) of the Green function may be expanded in a power series. The final result, which may be found in References 6 and 5, is a double series with positive powers of X and negative powers of Y:

$$F(X, Y)=2\sum_{n=0}^{\infty}\frac{(-X^2/4)^n}{(n!)^2}\left(\sum_{m=1}^{2n}\frac{(m-1)!}{Y^m}-e^{-Y}\mathrm{Ei}(Y)\right), \tag{19}$$

where $\mathrm{Ei}(Y)$ is the exponential integral. Equation (19) is used in the domain $0\leq X\leq 5$, $X/Y\leq 0{\cdot}5$ with $n=9$ terms.

3.4. Haskind integral representation

For values of X and Y in the interior domain between the lines $X=0{\cdot}5Y$, $Y=2$ and $X=5$ none of the previous three expansions can be used to evaluate the Green function. For this case the method developed by Telste and Noblesse[5] is appropriate. Briefly, this method may be described

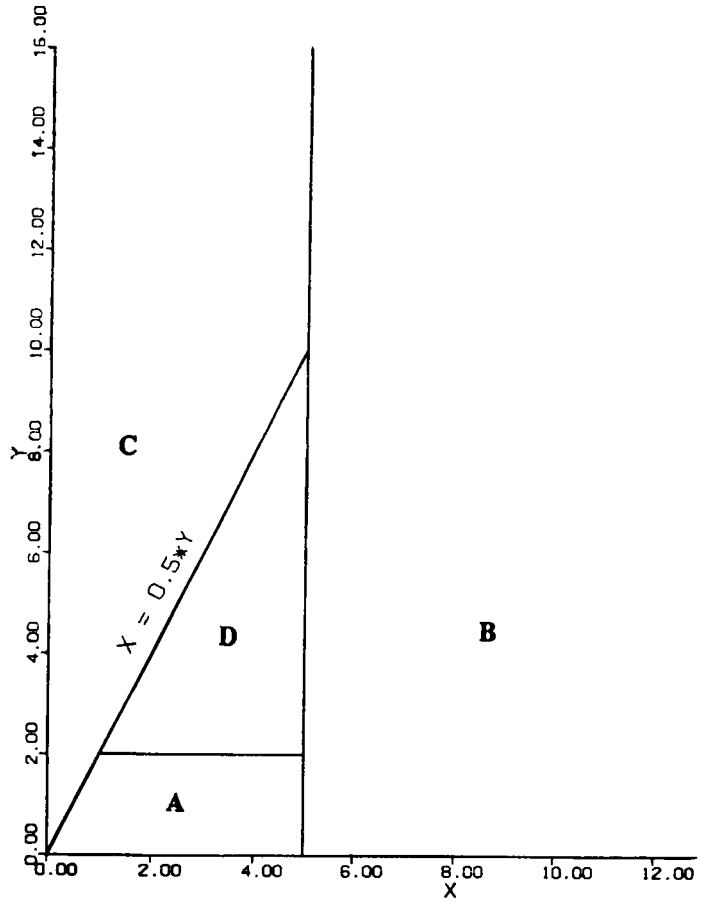

Figure 1. Domains of applicability of each algorithm for 6D accuracy evaluation of the Green function

Table I. Number of terms required in (17) for 6D absolute accuracy

	x				
Y	5	8	10	15	20
0·1	3	3	3	3	3
0·5	4	3	3	3	3
1	6	4	4	3	3
2	9	6	6	5	3
5	13	9	7	6	5
10	13	10	9	7	5
15	9	8	7	5	5
20	6	5	4	4	4

as follows. In the integral representation (13) change the variable of integration by setting $t = X\tau$ and expand the term $(1+\tau^2)^{-1/2}$ in a polynomial series. The resulting expressions may then be integrated analytically to give an approximation for the Green function.

3.5. Summary

In the previous subsections simple and efficient methods for evaluating the Green function have been described. The main advantage of these expansions is their simplicity and ease of numerical implementation. Only the first few terms in each expansion are necessary for 6D absolute accuracy. The domain of applicability of each algorithm is shown in Figure 1. More specifically, in region A, which is bounded by the lines $Y=2$, $X=0{\cdot}5Y$ and $X=5$, the Neumann series (15) is used with four to 15 terms depending on the values of X and Y. In region B, which is defined by $X \geq 5$, the expansion (17) is used with the number of terms given by Table I. No more than 13 terms are necessary for 6D accuracy. The expansion (19) is used in the domain $0 \leq X \leq 5$, $X/Y \leq 0{\cdot}5$ with nine terms. Finally, the Haskind integral representation is used in the interior domain bounded by the lines $X=0{\cdot}5Y$, $Y=2$ and $X=5$. The computing time is between 2×10^{-5} and 2×10^{-4} on an IBM 3090 machine for all algorithms considered.

4. SUPPRESSION OF THE IRREGULAR FREQUENCIES

In the vicinity of the irregular frequencies the algebraic system (8) becomes ill conditioned and the solution exhibits large numerical errors. There exist many methods of removing the irregular frequencies, some of them motivated from analogous methods used in acoustics where a similar non-uniqueness problem occurs. A survey of different methods may be found in Reference 14. Roughly speaking, the effectiveness of each method is judged using two criteria: (a) the method must be robust and general enough to handle three-dimensional problems involving bodies of arbitrary shape in all degrees of freedom; (b) the extra computational effort must be small when compared to that of the original boundary integral method.

In the present work, in order to eliminate the influence of the irregular frequencies, the M algebraic equations (8) are supplemented by the first N null-field equations. The result is an overdetermined system of $M+N$ equations for the M unknown potential strengths $(\phi_j)_m$ which is solved by a least squares procedure. In Reference 22 an analogous procedure was adopted for the solution of exterior acoustics problems without any numerical examples. It is possible to extend the analysis of Reference 22 and show that the interior potential ϕ_i and its first $N-1$ partial derivatives vanish at the origin. From the shape of the eigenmodes corresponding to each irregular frequency it follows that the first N irregular frequencies will be eliminated and as a result the method produces a unique solution for wave numbers $\nu < \nu_{N+1}$ (ν_{N+1} is the $(N+1)$th irregular wave number). Therefore, given an estimate of the $(N+1)$th irregular wave number (see e.g. Reference 23), the present method will give accurate results for all $\nu < \nu_{N+1}$. In practice the method can perform much better, because if, say, one uses only a single extra equation, one enforces $\phi_i = 0$ at the origin. Therefore in addition to the first irregular frequency in heave all the irregular frequencies for which $\phi_i \neq 0$ at the origin are suppressed. A similar argument can be used for more additional equations where the partial derivatives of ϕ_i are forced to vanish at the origin.

The present method is related to the combined boundary integral equation method (CBIEM) proposed in Reference 14, where the solution of the integral equation is supplemented by the requirement that the interior potential be zero at certain interior points. The selection of these interior points is arbitrary and special care must be exercised so that they do not coincide with the nodes of the interior eigenmode, because then the method fails. The effectiveness of the present method stems from the fact that it removes the arbitrariness in selecting the number and location

of these interior points. Instead, given an estimate of the location of the irregular frequencies, it provides a definite rule for the number of extra equations that must be used to guarantee uniqueness. Especially in the high-frequency range, where the number of nodes in the interior eigenmode increase, the present method is believed to have a clear advantage.

Regarding the additional computational effort required, the computational penalty consists of evaluating NM extra influence coefficients and solving a system of $N+M$ equations with M unknowns as opposed to an $M \times M$ system. Although a precise estimate of the extra CPU requirement has not been obtained, the CPU requirement for the modified method should not exceed $(M+N)/M$ as a percentage of the original method.

As a conclusion it is believed that the present method is effective in the spirit of the two criteria set at the beginning of this section, i.e. it is general enough to handle problems involving bodies of arbitrary shape and at the same time the extra computational cost is modest.

5. NUMERICAL RESULTS

To illustrate the performance of the numerical method, the added mass and damping coefficients for a sphere and a vertical circular cylinder have been computed. Since both bodies have two planes of symmetry, only one-quarter of the underwater surface needs to be discretized. Figure 2 shows the non-dimensional added mass and damping coefficients for a heaving sphere versus the non-dimensional wave number νR, where R is the radius of the sphere. 101 plane quadrilateral panels were used to discretize one-quarter of the underwater surface. The asterisks are results obtained by Barakat[24] using the method of multipoles. For a sphere in heave the first irregular wave numbers are $\nu_1 R = 2{\cdot}56$ and $\nu_2 R = 5{\cdot}52$. The dashed line results were obtained from the numerical solution of the system of equations (8). In the vicinity of the irregular frequencies substantial numerical errors occur. If, however, this system of equations is supplemented by the first symmetric null-field equation, the resulting solution, which is the solid line in Figure 2, is free from the irregular frequency problems for all wave numbers. For this particular example only the first symmetric null-field equation which enforces $\phi_i = 0$ at the origin is sufficient to guarantee uniqueness given that all interior eigenmodes corresponding to the irregular frequencies have $\phi_i \neq 0$ at the origin. The additional computational time introduced is very small since only M additional coefficients must be evaluated and the linear system of equations is augmented by one equation.

Figure 3 shows the non-dimensional added mass and damping coefficients for a sphere in surge versus the non-dimensional wave number. Again 101 plane quadrilateral panels were used to discretize the sphere surface. The asterisks are results computed by Hulme[25] using ring sources and a one-dimensional integral equation. As in the case of heave, the numerical solution of the system (8), represented by the dashed curve in Figure 3, suffers from large numerical errors in the vicinity of the irregular frequencies. To correct this problem, the system of equations is supplemented by the first non-symmetric null-field equation which enforces $\partial\phi_i/\partial r$ at the origin. The resulting solution, which is the solid line in Figure 3, is free from the irregular frequency problems and agrees well with the results obtained by Hulme[25] for all wave numbers.

Figure 4 presents the non-dimensional added mass and damping coefficients for a right circular cylinder in heave. The radius-to-draft ratio $R/T = 2$. The asterisks are results of Breit *et al.*[26] obtained by using ring sources and a one-dimensional integral equation. The results obtained by the present three-dimensional theory were obtained using a total of 113 panels consisting of a cosine spacing of 49 panels on the bottom and 48 panels on the side of a one-quarter body. In order to model the flow properly around the sharp corner where the bottom meets the side, small panels at an angle of 45° were placed. For this particular body the irregular wave numbers are

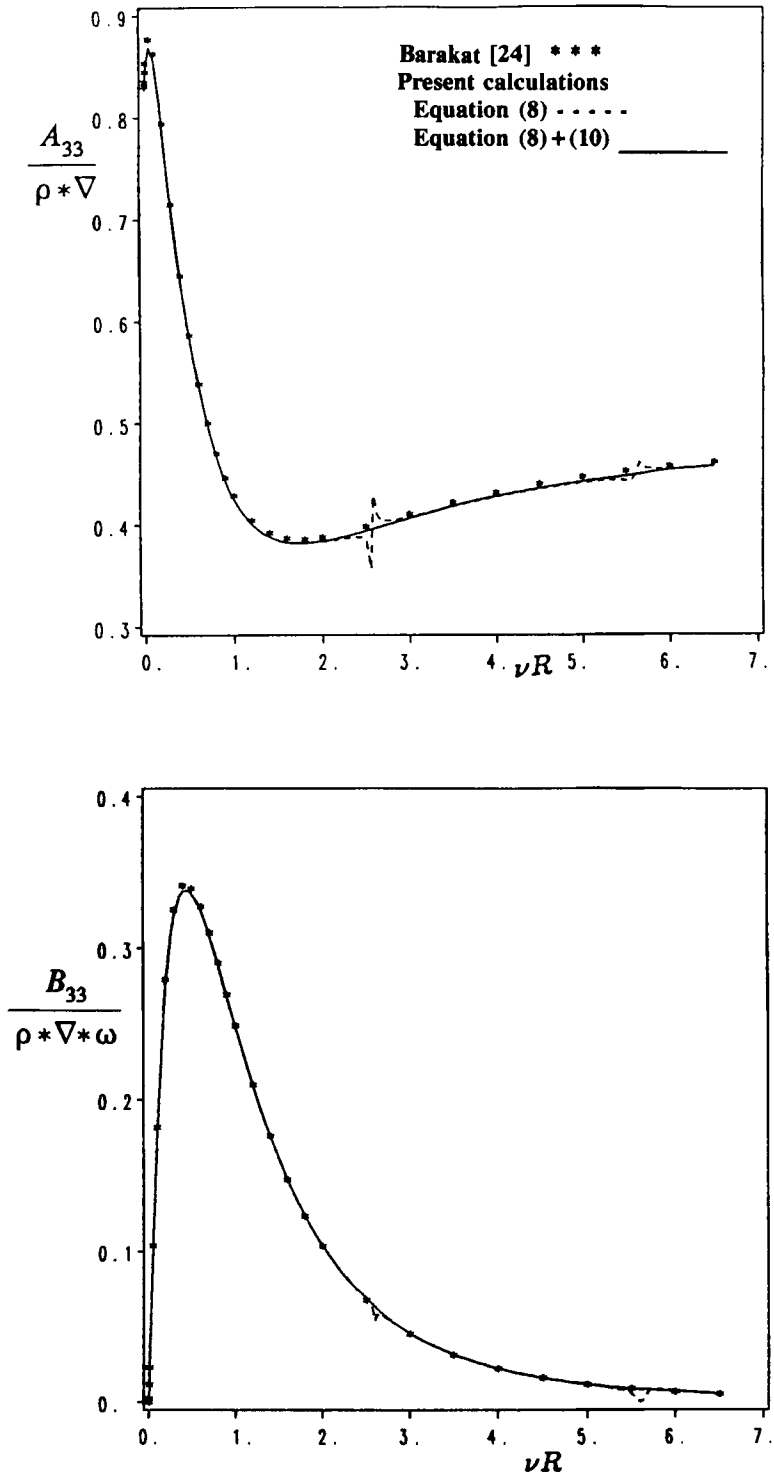

Figure 2. Heave added mass and damping coefficients for a sphere

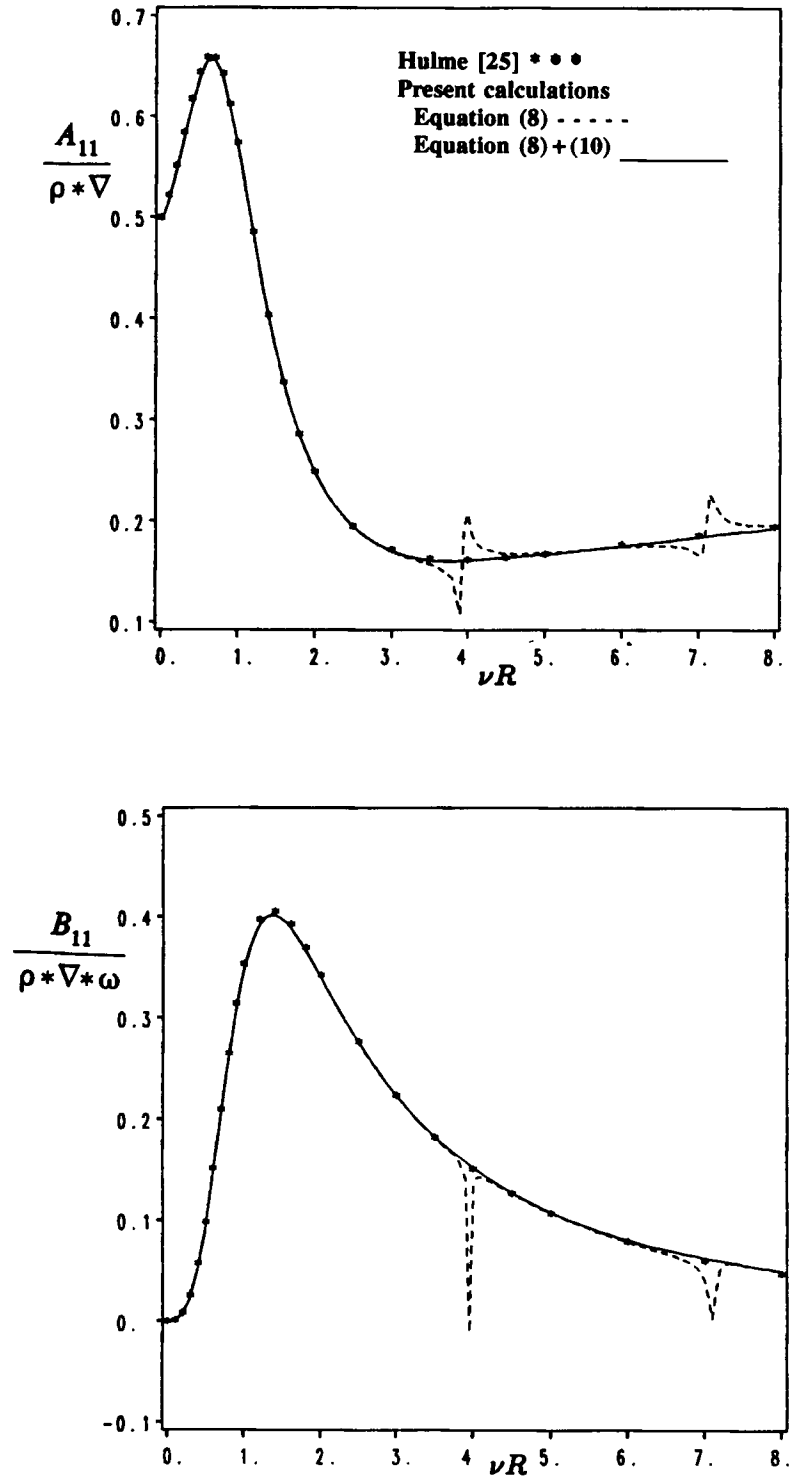

Figure 3. Surge added mass and damping coefficients for a sphere

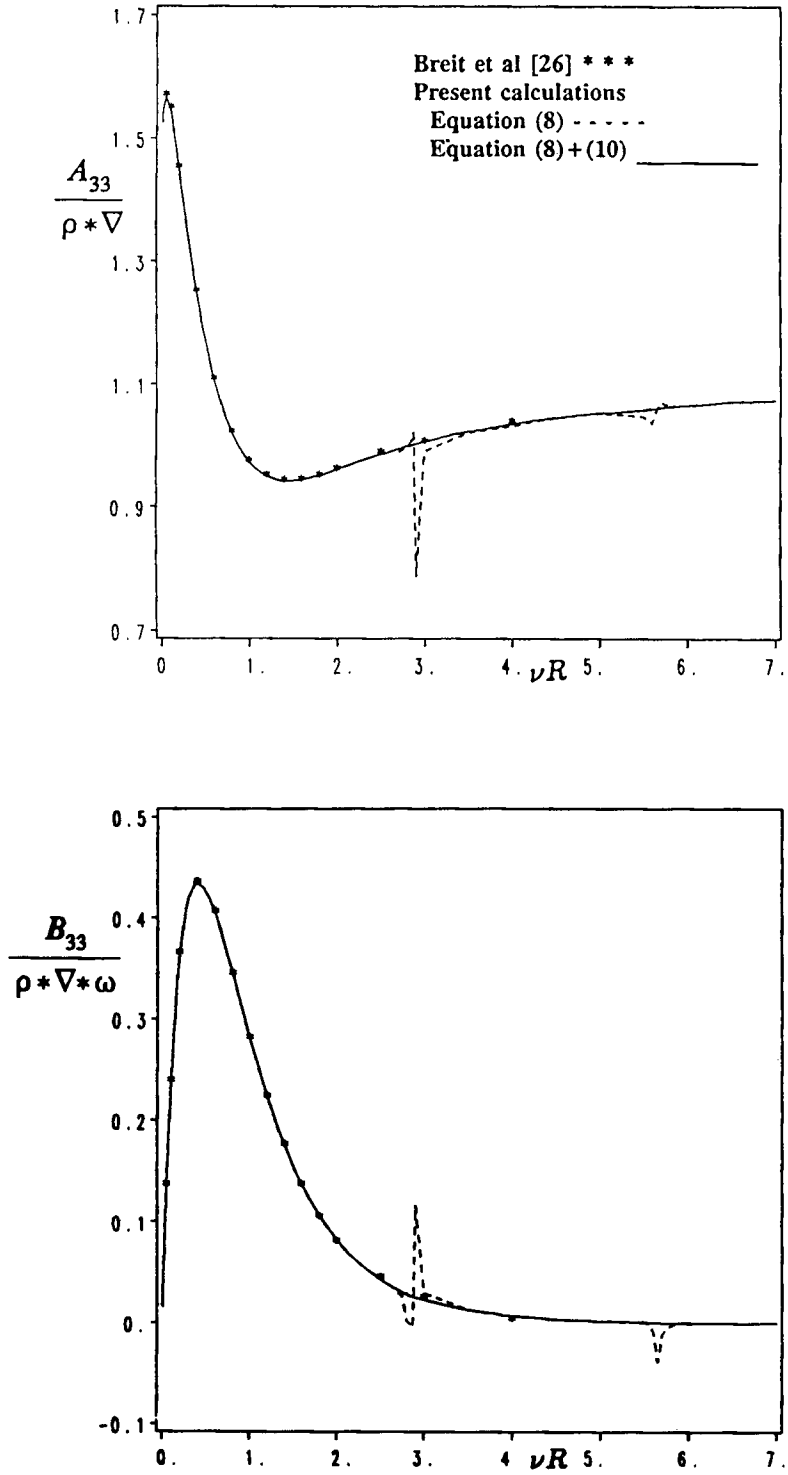

Figure 4. Heave added mass and damping coefficients for a right circular cylinder

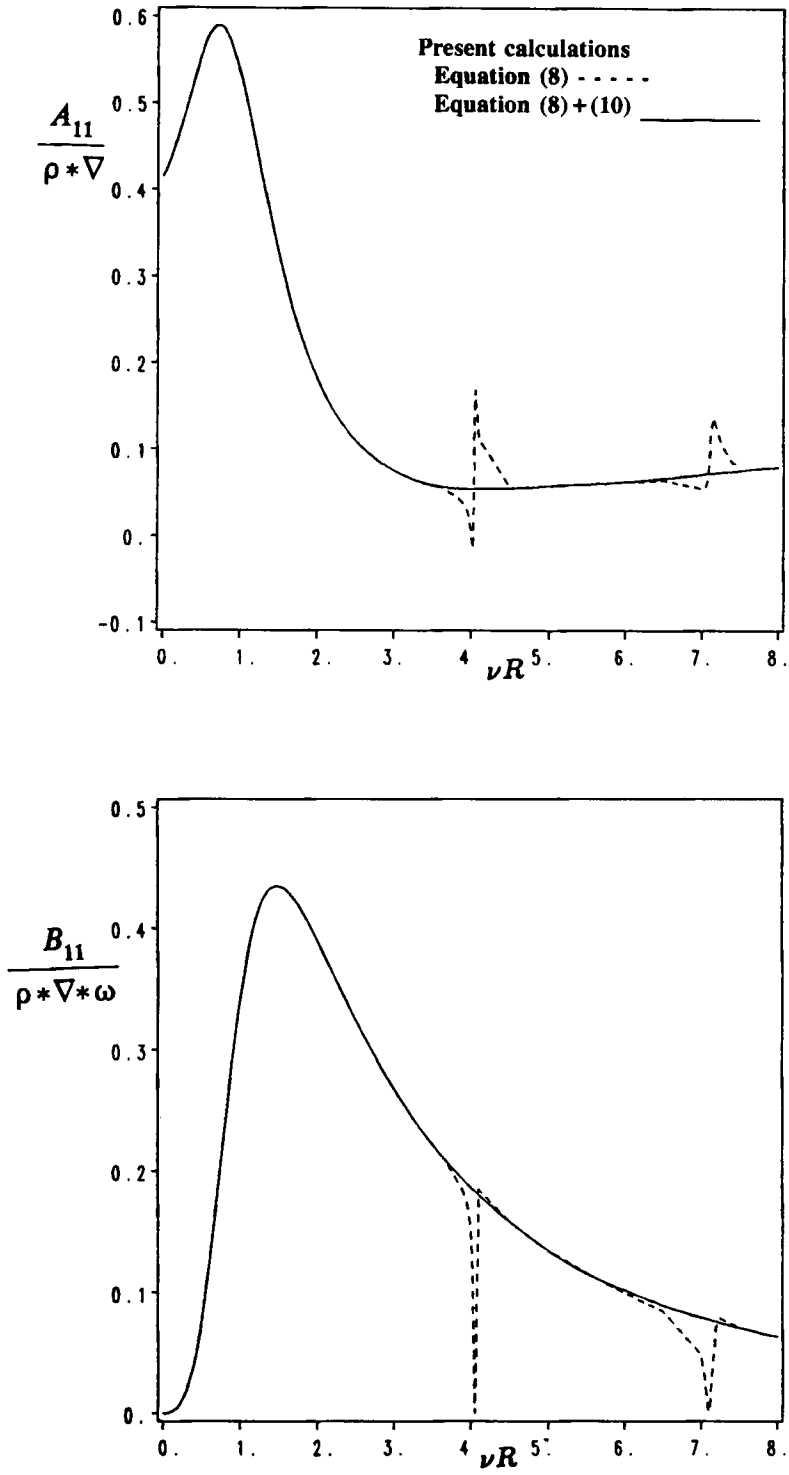

Figure 5. Surge added mass and damping coefficients for a right circular cylinder

given explicitly[11] as $\nu_M R = K_M R \coth(K_M T)$, where $K_M R$ are the roots of the Bessel function J_0. As in the case of a sphere, the numerical solution of the system (8) represented by the dashed curve in Figure 4 exhibits large errors close to the irregular frequencies. If, however, the first symmetric null-field equation is used, the solution does not exhibit any irregular frequency effect.

Finally, in Figure 5 the non-dimensional added mass and damping coefficients for a right circular cylinder in surge are shown. As in the previous cases, the system (8) overdetermined with the first non-symmetric null-field equation is free from the irregular frequency effects for all wave numbers.

6. CONCLUSIONS

A boundary integral equation method solving three-dimensional free surface problems has been developed. Efficient algorithms for the numerical evaluation of the Green function have been presented. In order to remove the numerical difficulties with the irregular frequencies, the boundary integral equation is overdetermined with the first N null-field equations. The resulting solution is free from the first N irregular frequencies.

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